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ON THE DETERMINATION OF HORIZONTAL FORCES A FLOATING ICE PLATE --ETC(U)
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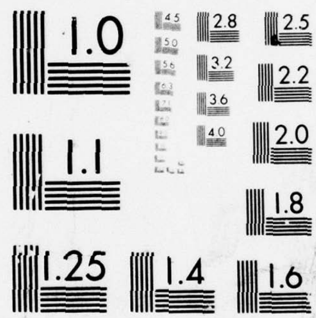

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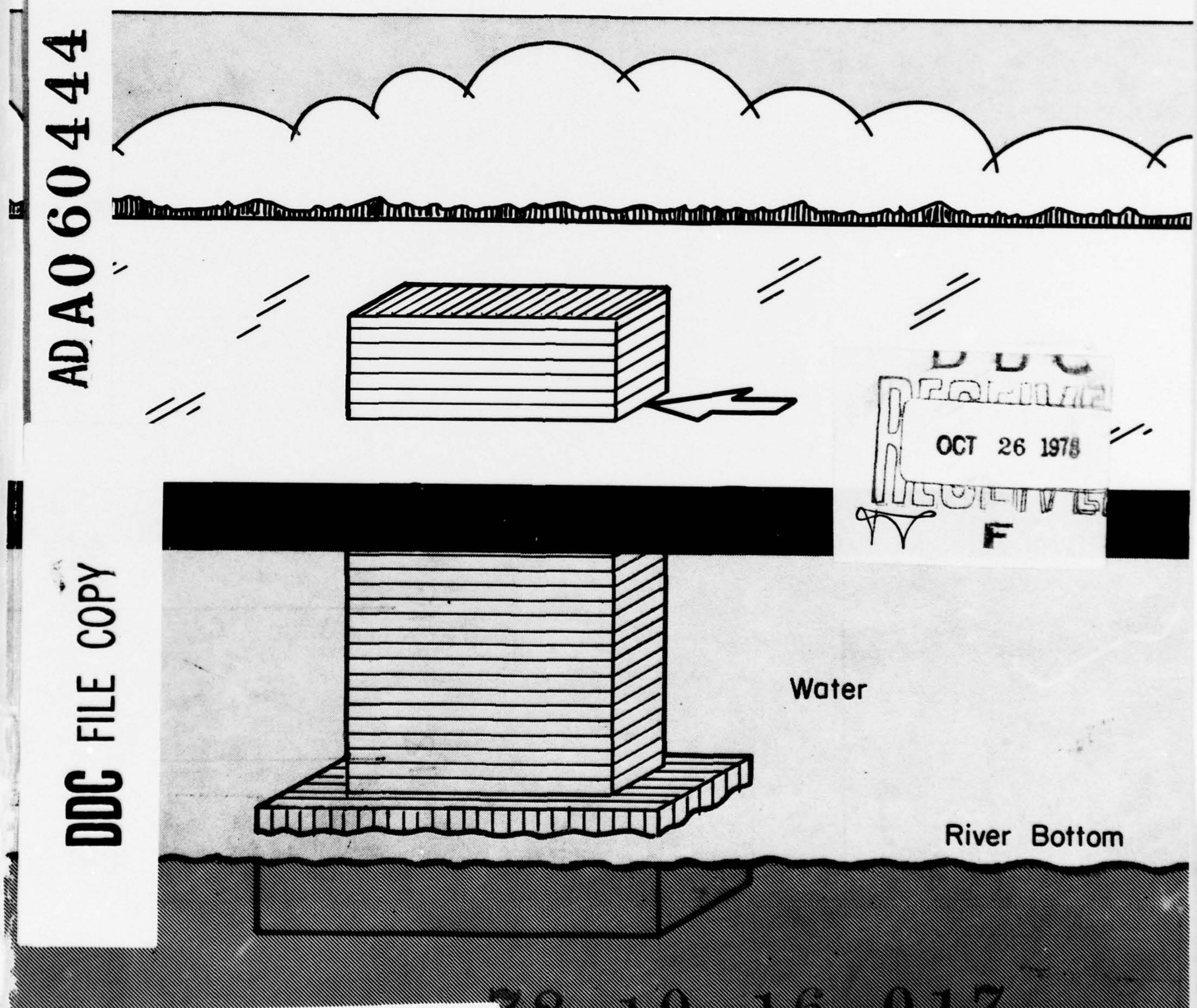
On the determination of horizontal forces a floating ice plate exerts on a structure



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Arnold D. Kerr

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This report first discusses the general approach for calculating the horizontal forces an ice cover exerts on a structure. Ice force determination consists of two parts: (1) the analysis of the in-plane forces assuming that the ice cover remains intact and (2) the use of a failure criterion, since an ice force cannot be larger than the force capable of breaking up the ice cover. For an estimate of the *largest* ice force, an *elastic* plate analysis and a failure criterion are often sufficient. A review of the literature revealed that, in the majority of the analyses, it is assumed that the failure load is directly related to a "crushing strength" of the ice cover. However, observations in the field and tests in the laboratory show that in some instances the ice cover fails by buckling. This report reviews the ice force analyses based on the *buckling* failure mechanism and points out their shortcomings. The report then presents a new method

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of analysis which is based on the buckling mechanism.

PREFACE

This report was prepared by Dr. Arnold D. Kerr, Visiting Professor, Department of Civil Engineering, Princeton University, for the U.S. Army Cold Regions Research and Engineering Laboratory. G.E. Frankenstein of CRREL was Technical Monitor of the program covered by this report.

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Technical review of this report was performed by Dr. Donald E. Nevel and F. Donald Haynes of CRREL.

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ILLUSTRATIONS

ON THE DETERMINATION OF HORIZONTAL FORCES A FLOATING ICE PLATE EXERTS ON A STRUCTURE

Arnold D. Kerr

INTRODUCTION AND STATEMENT OF PROBLEM

When a river, lake, or part of a sea freezes over, the ice cover formed is often in contact with a variety of structures, such as bridge piers, dams, waterpower stations, and offshore drilling platforms. Subsequently, any change in the position of the ice cover (by changing the water level, by moving the water base underneath it or the air above it, or by changing the air temperature) creates forces between the structures and the ice cover formed.

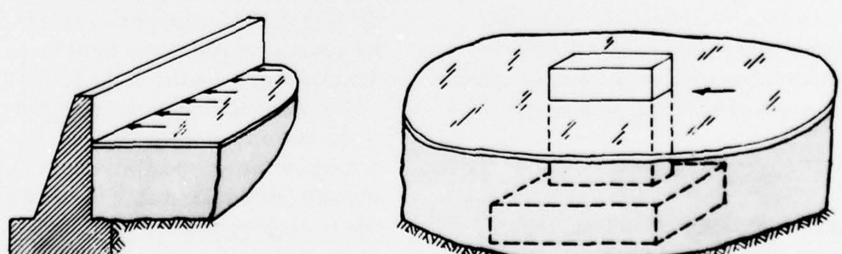
Thus, for a rational design of a structure that comes in contact with a floating ice cover, the engineer needs to know the forces the ice will exert on this structure, especially the largest possible force.

Early descriptions of problems of this type were given by Barnes[6] and by Komarovskii.[16] The effort and progress made in this field during the following few decades were very modest, as evidenced by the results presented at the American Society of Civil Engineers Symposium on Ice Pressure Against Dams in 1954.

During the past decade, the problem of ice forces has gained in importance, partly because of the discovery of oil in the northern regions and the resulting increased activity in this area. The intensified research effort that has followed is evidenced by the recent increase of conferences on this subject, such as the Conference on Ice Pressures Against Structures[8] held in 1966, the Ice Seminar[11] held in 1968, the International Conferences on Port and Ocean Engineering under Arctic Conditions[13] held in 1971, 1973 and 1975, and the IAHR/AIRH Symposia on Ice and Its Action on Hydraulic Structures[12] held in 1970, 1972, and 1975. In spite of all these efforts, to date no reliable methods have been developed for predicting the ice forces an ice cover may exert on a structure.

The analyses of these ice forces are usually grouped into several categories, for example: 1) analyses of forces caused by a rise or drop in the water level, 2) analyses of forces caused by a structure constraining the movement of the ice cover in the horizontal plane, and 3) analyses of forces caused by the impact of a moving ice flow and a structure. This report discusses problems of the second category. Examples of such problems are shown in Figure 1.

Since ice is a viscoelastic material, the forces an ice cover exerts on a structure are time dependent. The



a. Ice cover edge pressure against rigid wall.

b. Ice cover pressing against an isolated structure.

Figure 1. Structures which constrain the movement of the ice cover.

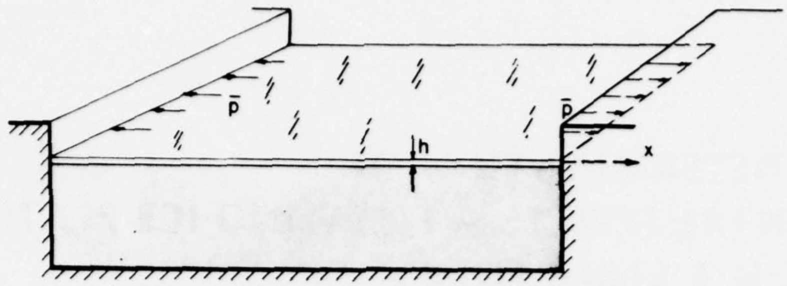


Figure 2. Ice cover constrained by two parallel rigid walls. $\bar{p}$ = ice pressure against wall (per unit length of wall) and h = thickness of ice cover.

study of these forces, caused by constrained thermal expansions, was initiated by Royen in 1922. The analytical and test results of Royen, as well as the more recent results of a number of other investigators, were reviewed by Korzhavin[17] (chapter XIV) in 1962, by Drouin[7] in 1966, and by Michel[18] in 1970. Other attempts to solve this problem were presented by Nevel[19] in 1968 and by Panfilov[21] in 1965.

The conclusion of the survey by Drouin[7] (still valid presently) was that the determination of ice forces as a function of time is not yet solved satisfactorily. The relevant results of references 19 and 21 are not conclusive either. A major difficulty in solving this problem is the absence of appropriate constitutive equations for the ice cover. It should also be noted that, even if expressions which determine the ice forces were available, the magnitude of these forces may sometimes be limited by a possible failure (crushing or buckling) of the ice cover. This aspect will be discussed later.

For the design of a structure that is in contact with an ice cover, it is often sufficient to know the *largest* force the ice cover may exert on the structure. This approach simplifies the necessary analyses.

To demonstrate the essential aspects of this ice force problem, consider, as an example, an ice cover constrained by two parallel "rigid" walls, as shown in Figure 2.

Assume that (at a time $t = 0$) the temperature of the ice plate is instantaneously raised by a constant value, ΔT°. Then, the corresponding axial compression force $\bar{p}_0$ (per unit length of wall) is approximately

$$\bar{p}_0 = Eh\alpha\Delta T^\circ. \qquad (1)$$

Equation 1 is based on the assumption that at the instant of the temperature rise ($t = 0$) the ice obeys Hooke's law, that Young's modulus E is a value averaged across the plate thickness, and that the coefficient of linear expansion α is also an average value which doesn't vary noticeably during the temperature increase ΔT°.

Because of the viscoelastic response of the ice cover, the pressure $\bar{p}$, for $t > 0$, is smaller than the $\bar{p}_0$ value given in eq 1:

$$\bar{p}(t) < \bar{p}_0. \qquad (2)$$

A similar argument leads to the conclusion that eq 2 is also valid for the case when the temperature rise is not instantaneous, but proceeds gradually, as is the case in the field. Thus, eq 1 yields the largest possible axial compression force due to ΔT°. This suggests that, for many engineering problems, it may be sufficient to determine the forces acting on a structure by assuming that the ice cover responds elastically.

At this point, it should be noted that the force estimate given by eq 1, or the more accurate expression which takes into consideration the viscoelastic response of the ice cover, is valid as long as the ice cover does not fail. Thus, beyond a certain compression force, eq 1, or its viscoelastic equivalent, yields too high values for the ice force and it has to be bounded by a failure criterion. This criterion is obtained by noting that *an ice force cannot be larger than the force capable of breaking up the ice cover.* Thus, the needed bound for in-plane compression forces is established by the determination of the intensity of the force at which the ice cover fails. Note that this criterion is valid for the determination of the largest ice forces that may be due to a change in temperature or due to any other reason, such as the movement of the water in the horizontal direction.

Many published analyses are based on this notion. In the majority of these publications, it is assumed that this failure load is directly related to a "crushing strength" of the ice; that is, the mechanism of failure is due to crushing or splitting of the floating ice plate in the immediate vicinity of the structure. A review of the relevant literature up to 1962 is given by Korzhavin.[17] A more recent use of this approach is presented by Afanasiev,[1,2] Assur,[5] Michel,[18] Schwarz,[25] and Shadrin and Panfilov.[26]

In laboratory and field tests it was observed, however, that, for relatively thin ice plates (compared with the width of contact of plate and structure), the ice cover failed by buckling in the vicinity of the contact area.[3 15 20 24]

The published analyses of ice forces on isolated structures, which are based on the buckling failure mechanism, are not conclusive, however. The purpose of this report is first to review these few attempts and then to develop an improved method of analysis for the determination of the largest force that a relatively thin ice cover may exert on an isolated structure. The proposed analysis is based on the buckling mechanism of the floating ice plate.

REVIEW OF RELEVANT ANALYSES AND TESTS

An early attempt to determine analytically the buckling force for an ice plate was presented by Albenga[4] in 1921. However, because he neglected the effect of the liquid base, he obtained results that do not apply to floating ice covers.

On the basis of observations in the field, in 1947 Rose[24] (p. 883) stated: "... that arching and buckling of an ice sheet does occur but that there is a limiting thickness of about 1 ft beyond which buckling will rarely occur." He also claimed (p. 880): "... for ice sheets less than 1 ft thick, the critical buckling load is the limiting factor governing the ice pressure on a dam, whereas for very thick ice sheets (about 2 ft or more), the crushing strength pressure" is the limiting factor. No analysis was given for the determination of the buckling load.

In 1962, Shadrin and Panfilov[26] presented an analysis for the semi-infinite ice cover whose straight edge is pressing against a rigid wall, as shown in Figure 1a and Figure 2. The authors assumed that the type of ice cover failure depends on the thickness of the ice cover, namely, buckling for thin covers and local crushing at the contact area for thick covers. This assumption is in agreement with the observations made by Rose,[24] as described above.

Shadrin and Panfilov[26] expressed the failure load (per unit length) as

$$\bar{p}_f = \sigma_{cf} h \tag{3}$$

where σ_{cf} is a compression strength and h is the thickness of the ice cover.

For relatively thin covers, the buckling load p_b was determined from the differential equation

$$D \frac{d^4 w}{dx^4} + \bar{p} \frac{d^2 w}{dx^2} + \gamma w = 0 \tag{4}$$

and the boundary conditions

$$\left.\begin{aligned} w(0) &= 0 \\ \left.\frac{d^2 w}{dx^2}\right|_0 &= 0 \end{aligned}\right\} \tag{5}$$

where $w(x)$ is the vertical deflection at x and at any value $-\infty < y < \infty$, $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of the cover, and γ is the specific weight of the liquid base. The solution of the above formulation yields

$$\bar{p}_b = 2\sqrt{\gamma D}\ . \tag{6}$$

The above expression for $\bar{p}_b$ is also valid when the boundary at $x = 0$ is clamped,[9] as may often be the case in the field.

Next, eq 3 and 6 are compared by noting that, according to tests, if the corresponding

$$\bar{p}_b < \bar{p}_f$$

the ice cover fails by buckling and if

$$\bar{p}_f < \bar{p}_b$$

the ice cover fails by crushing. Substituting eq 3 and 6 in the first inequality, it follows that the ice cover will fail by buckling when

$$h < \frac{3(1-\nu^2)\,\sigma_{cf}^2}{\gamma E}\ . \tag{7}$$

As an example, if $\nu = 0.34$, $\gamma = 0.001$ kg/cm^3, $E =$ 40,000 kg/cm^2, and $\sigma_{cf} = 20$ kg/cm^2, then according to eq 7 the ice cover will buckle when

$$h < 27 \text{ cm} \cong 10.5 \text{ in.}$$

This result agrees reasonably well with the field observations reported by Rose.[24]

According to the above analysis, when inequality (7) is satisfied, the largest force that the ice cover may exert on the long rigid structure is the buckling load $\bar{p}_b = 2\sqrt{\gamma D}$; otherwise, the largest force is $\bar{p}_f = \sigma_{cf} h$.

Next, the calculation of the largest force an ice cover exerts on a ship's hull, presented by Kheishin[15] in 1961 is reviewed; this is a problem closely related to the determination of ice forces on isolated structures. Kheishin

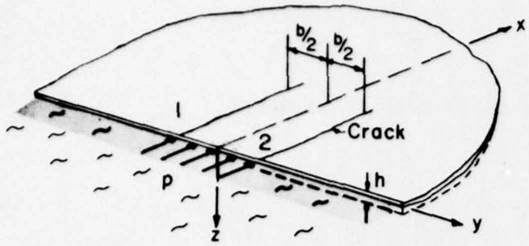


Figure 3. Analytical model used by Kheishin.

reduced this problem to the determination of the pressure p = constant at which a floating elastic semi-infinite plate, shown in Figure 3, will fail. He based his analysis on field observations that ice cover failures often occur in two stages: the formation of cracks which emanate from points 1 and 2 (Fig. 3), followed by the buckling of the inner part of the ice cover, which is formed by these cracks.

For his analysis, Kheishin[15] assumed that the cracks are initiated by the largest shear stress along the edge

$$(\sigma_{xy})_{max} = \sigma_{xy}\,(0, \pm b/2) = p/\pi h. \tag{8}$$

Choosing for the shear strength of the (sea) ice cover σ_{sf} = 30 tons/m^2, and using the equation

$$(\sigma_{xy})_{max} = \sigma_{sf} \tag{9}$$

as a *crack criterion*, he obtained from eq 8 the pressure intensity that causes the formation of the cracks as

$$p_{sf} \cong 94\,h \quad \text{tons/m} \tag{10}$$

where h is in meters.

To simplify the buckling analysis, Kheishin[15] assumed that the two cracks which emanate from the points (0, ± b/2) are *parallel* to the x-axis and that the formed ice strip buckles when

$$p_b\Big|_{cyl\ bend} = \sqrt{\gamma D}\,. \tag{11}$$

Assuming $\nu = 0.36$, γ = 1 ton/m^3, and E = 40,000 tons/m^2, the above equation reduces to

$$p_b\Big|_{cyl\ bend} = 62\,h\sqrt{h} \quad \text{tons/m} \tag{12}$$

where h is in meters.

Comparing eq 10 and eq 12, Kheishin[15] obtained that for $0 < h < 2.2$ m,* which is the range of practical interest, the inequality

$$p_{sf} > p_b\Big|_{cyl\ bend} \tag{13}$$

holds (although, according to Kheishin,[15] $p_{sf} < p_b$, p_b being the buckling load of the uncracked plate). Based on observations in the field, that an ice cover cracks prior to buckling, Kheishin concluded that the largest ice force is given by p_{sf}, in eq 10. According to the above analysis, after the plate cracks, buckling of the inner strip takes place immediately, since

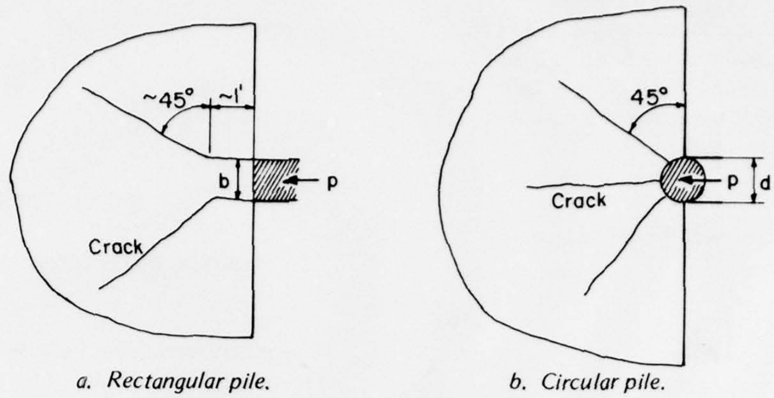
$$p_b\Big|_{cyl\ bend} < p_{sf}. \tag{14}$$

The above method is based on a number of questionable assumptions, such as that eq 9 is a valid criterion, that the cracks are parallel and normal to the free edge, etc. Note also that p_b given in eq 11 is the critical buckling pressure when the edge is free to rotate and displace vertically,[9] which is usually not the case when the ice cover is in contact with a fixed vertical structure or with the hull of a ship. Other shortcomings of eq 11 are that the ice strip formed by two cracks that emanate from points 1 and 2 is not of constant width and that the strip is constrained along the entire length of the formed edges by the surrounding ice cover. Also, according to Nevel et al.,[20] who conducted many closely related tests, the formation of radial cracks is *not* followed immediately by ice cover failure. Therefore, the results obtained by Kheishin,[15] which are also reproduced in the book by Popov et al.,[23] should be used with caution.

In 1971, Afanasiev et al.[3] published results of an extensive laboratory test program in which a number of thin, floating ice plates ($2.6 \leqq h \leqq 3.4$ cm) were subjected to a horizontal force by pressing a circular or a rectangular pier-model against the free edge of the plates. According to their results, when $b/h > 4$ and the ice cover was not made more rigid by the proximity of the basin walls or the presence of simulated ridges, the ice plate failed by buckling. During these tests, it was observed (ref. 3, p. 64) that, at a certain intensity of the load, radial cracks formed in the plate. At a further increase of the load, the ice plate lifted along a nearly circular curve at a distance of about $14 \times h$ from the pier. This was followed by a breakdown of the ice field.

Afanasiev et al.[3] then presented an analysis for the semi-infinite ice plate pressing uniformly against a rigid wall, which is similar to the one given by Shadrin and Panfilov[26] discussed previously. The results they obtained are also similar: for fresh water ice the ice cover buckles when $h < 30$ cm, and for salt water ice

* Note that in the previous example (Shadrin and Panfilov[26]) E = 400,000 tons/m^2 was used. For this E-value the corresponding interval is $0 < h < 0.23$ m.



a. Rectangular pile. *b. Circular pile.*

Figure 4. Observed crack patterns for different shapes of model pier.

when $h < 20$ cm. Thus, when these inequalities are satisfied, the buckling load is the largest force the ice cover can exert against a rigid wall.

In 1972, Nevel et al.[20] presented results of extensive ice force tests conducted at CRREL. The test set-up was similar to the one described by Afanasiev et al.,[3] except that the basin used (6.3x6.3 m) and the thickness of the plates tested ($6.5 \leq h \leq 22.4$ cm) were larger. Their description of the test results is also fuller. The results of tests with *rectangular* piers showed that for an average ice temperature of −4.3°C and $h \cong 12$ cm, for $b/h > 2$ the ice plate failed by buckling. For *circular* piers, instead of the b-value, they used the pier diameter d. From the test results presented, it follows that for cold ice (−4.0°C) the ice plate buckled for $d/h > 4.85$, whereas for warmer ice (−2.7°C) the ice plate buckled for larger ratios of d/h: $d/h > 8.2$. In some of these tests, it was observed that, as the load increased, cracks occurred, as indicated in Figure 4. For rectangular piles (Fig. 4a), the cracks formed a funnel-shaped plate which buckled in the wider part of the plate.

According to the above test results,[3 20] for rectangular piers, a thin ice plate may fail in buckling for $b/h > 2$ and for circular piers for $d/h > 5$, ranges of practical interest. Thus, for a variety of isolated structures and relatively thin ice covers, the largest ice forces appear to be related to the buckling load of the ice cover and not to the local "crushing strength."

The results of the reviewed analyses presented by Shadrin and Panfilov[26] and by Afanasiev et al.[3] for the determination of the largest force an ice cover may exert on a *long vertical wall* (for example, a long dam) are in agreement with observations in the field, regarding occurrence of buckling failures. Therefore, it is reasonable to expect that if the material parameters are chosen properly, the largest ice force for problems of this type can be estimated using the analyses discussed: eq 4 to 7.

However, for the determination of the largest ice forces on *isolated structures*, based on the buckling failure mode, only the analysis of Kheishin[15] seems to be available. As shown above, his analysis is based on questionable assumptions. According to Kheishin, the failure load is the load at which the plate cracks, since buckling takes place *immediately* after the cracks form; but this is not substantiated by the observations described by Afanasiev et al.[3] and Nevel et al.[20]

These findings suggest the need for an improved analysis for the determination of the largest forces an ice cover exerts on isolated structures, based on the buckling failure mechanism. Such an analysis is presented in the following section.

DETERMINATION OF THE LARGEST ICE FORCE ON AN ISOLATED STRUCTURE

Preliminary remarks

According to the laboratory tests by Nevel et al.,[20] when a pier with a flat surface presses against the free edge of a floating ice plate (or vice versa), at a certain intensity of the in-plane load radial cracks emanate from the loaded region, as indicated in Figure 4a; a further increase of the load leads to more radial cracks and finally to the buckling of the cracked region. A similar observation was also made by Afanasiev et al.[3] These observations suggest that the largest force a relatively thin ice cover could exert on an isolated structure may be related to the buckling load of floating wedge-shaped plates.

The buckling analysis of a floating wedge

Consider a floating truncated wedge, as shown in Figure 5. It is assumed that the buckling load P_b may be determined, approximately, from the eigenvalue problem, consisting of the differential equation

$$[EI(x)w'']'' + Pw'' + k(x)w = 0 \qquad 0 < x < \infty \qquad (15a)$$

the boundary conditions at the contact area

$$\left.\begin{aligned} w(0) &= 0 \\ w''(0) &= 0 \end{aligned}\right\} \qquad (16a)$$

or

$$\left.\begin{aligned} w(0) &= 0 \\ w'(0) &= 0 \end{aligned}\right\} \qquad (16b)$$

and the regularity conditions at $x = \infty$.

In the above formulation, $w(x)$ is a vertical perturbation of the wedge axis at x. Furthermore,

$$\left.\begin{aligned} k(x) &= \gamma(b_0 + x\phi^*) \\ EI(x) &= D(b_0 + x\phi^*) \end{aligned}\right\} \qquad (15b)$$

where

$$\left.\begin{aligned} D &= \frac{Eh^3}{12(1-\nu^2)} \\ \phi^* &= 2 \operatorname{tg} \frac{\phi}{2} \end{aligned}\right\} \qquad (15c)$$

and γ is the specific weight of the liquid base. The term $(1-\nu^2)$ was inserted in the denominator of $EI(x)$ to account for plate action, as done by Papkovich[22] in connection with a related problem, recently discussed by Kerr[14] (p. 15).

Substituting eq 15b into eq 15a, results in

$$[D(b_0 + x\phi^*)w'']'' + Pw'' + \gamma(b_0 + x\phi^*)w = 0 \qquad 0 < x < \infty \qquad (15d)$$

a fourth order equation with variable coefficients. For $\phi^* = 0$ (i.e., for $\phi = 0$), the above equation reduces to the equation with constant coefficients used by Kheishin,[15] which is the same as eq 4.

Because of the nature of the variable coefficients, no exact close form solution is available. A numerical solution can be easily obtained using, for example, a

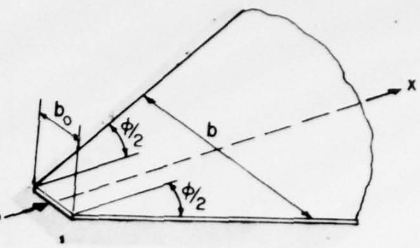


Figure 5. Floating truncated ice wedge.

finite difference scheme, because the buckling mode is restricted to the vicinity of the contact region at $x = 0$. However, the solution obtained will be valid only for the assumed set of parameters.

For the proposed method to determine P_{max}, an analytical expression that shows the dependence of the buckling load P_b on the various geometrical and material parameters of the floating ice wedge and the width of the structure is more suitable. Noting that the representation of the buckling of a cracked ice cover by the buckling of a floating wedge is at best an approximation, it appears sufficient to solve the wedge formulation approximately. In the following, such an analytical solution is obtained, at first, using the Stodola-Vianello method.[10]

For this purpose, eq 15d is rewritten as

$$w^{iv} + 4\kappa^4 w = -\frac{\phi^*}{b_0}\left[(xw_1'')'' + \frac{\gamma}{D} xw_1\right] - \lambda^2 w_1'' \qquad (17a)$$

where

$$\left.\begin{aligned} \kappa^4 &= \frac{\gamma}{4D} \\ \lambda^2 &= \frac{P}{Db_0}\,. \end{aligned}\right\} \qquad (17b)$$

Assuming for w_1 the expression

$$w_1(x) = A_1 \, e^{-\kappa x} \sin \kappa x \qquad (18)$$

and substituting it into eq 17a, a nonhomogeneous ordinary differential equation with constant coefficients for $w(x)$ is obtained. Its solution, which satisfies the boundary conditions of a simply supported edge, eq 16a, is

$$w(x) = A_1 \, e^{-\kappa x} \, [A \sin(\kappa x) + (\kappa A + B)x \sin(\kappa x) + Bx \cos(\kappa x)] \qquad (19a)$$

where

$$\left.\begin{aligned} A &= \frac{-\phi^*}{2\kappa b_0} \\ B &= \frac{P}{8D\kappa b_0}\,. \end{aligned}\right\} \qquad (19b)$$

The first eigenvalue is obtained by substituting eq 19a into the condition

$$\int_0^\infty w_1^2(x)\,dx = \int_0^\infty w_1(x)\,w(x)\,dx\,. \qquad (20)$$

Performing the necessary integrations and noting that $\lambda^2 = P/(Db_0)$, the approximate buckling load is obtained as

$$(P_b)_{\substack{\text{simply}\\ \text{supp}}} = 5.3\,D\kappa\,(\kappa b_0 + \phi^*). \qquad (21a)$$

If, instead of the boundary conditions for a simply supported end, the ones for a clamped end at $x = 0$ are used, the above procedure yields

$$(P_b)_{\text{clamped}} = 8D\kappa\,(2\kappa b_0 + \phi^*). \qquad (21b)$$

Note that in both cases the respective P_b expression consists of the sum of two terms:

$(D\kappa^2 b_0)$

$(D\kappa\phi^*)$

each multiplied by a constant coefficient. Thus, for the obtained P_b expressions, the different boundary conditions at $x = 0$ affect only these coefficients.

To check if the above two terms are representative of the (bifurcation) buckling solution for the compressed wedge, in the following the wedge problem is solved again using the energy method.

With the notation used above, the second variation of the total potential energy for the wedge problem shown in Figure 5 is

$$V_2 = \int_0^\infty \left(\frac{EI}{2}\,w''^2 + \frac{k}{2}\,w^2 - \frac{P}{2}\,w'^2\right) dx \qquad (22)$$

where EI and k are as given in eq 15b.

Assuming

$$w(x) = A\,e^{-\beta x}\sin\beta x \qquad (23)$$

as an approximate shape function, substituting it into V_2, and performing the necessary integrations, V_2 becomes

$$V_2 = \frac{A^2}{8\beta^2}\left[(6Db_0\beta^4 - P\beta^2 + \gamma b_0/2)\beta + \phi^*(2D\beta^4 + \gamma/2)\right]. \qquad (24)$$

If β is chosen *a priori* as the value which corresponds to the compressed semi-infinite plate,

$$\beta = \kappa' = \sqrt[4]{\gamma/D} = \sqrt[4]{4}\,\kappa\,, \qquad (25)$$

it follows that $V_2 = V_2(A)$. The condition $\delta V_2 = 0$ then reduces to

$$\frac{\partial V_2}{\partial A} = 0. \qquad (26)$$

The above equation yields

$$(6Db_0\kappa'^4 + \gamma b_0/2 - P_b\kappa'^2)\kappa' + \phi^*\,(2D\kappa'^4 + \gamma/2) = 0. \qquad (27)$$

Thus

$$P_b = 6D\kappa'\left[b_0\kappa'\left(1 + \frac{\gamma}{12D\kappa'^4}\right) + \phi^*\left(\frac{1}{3} + \frac{\gamma}{12D\kappa'^4}\right)\right] \qquad (28a)$$

and noting that $\gamma/(12D\kappa'^4) = 1/12$ and $\kappa' = \sqrt[4]{4}\,\kappa$, P_b reduces to

$$P_b = 3.5D\kappa\,[3.7b_0\kappa + \phi^*]. \qquad (28b)$$

If β is not fixed *a priori*, then $V_2 = V_2(A, \beta)$. The condition $\delta V_2 = 0$ then reduces to

$$\left.\begin{aligned} \frac{\partial V_2}{\partial A} &= 0 \\ \frac{\partial V_2}{\partial \beta} &= 0. \end{aligned}\right\} \qquad (29)$$

The first equation in (29) is identical to eq 26, and yields eq 28a with $\kappa' = \beta$:

$$P_b = 6D\beta\left[b_0\beta\left(1 + \frac{\gamma}{12D\beta^4}\right) + \phi^*\left(\frac{1}{3} + \frac{\gamma}{12D\beta^4}\right)\right]. \qquad (30)$$

The second equation in (29), after eliminating P_b by using eq 30, yields

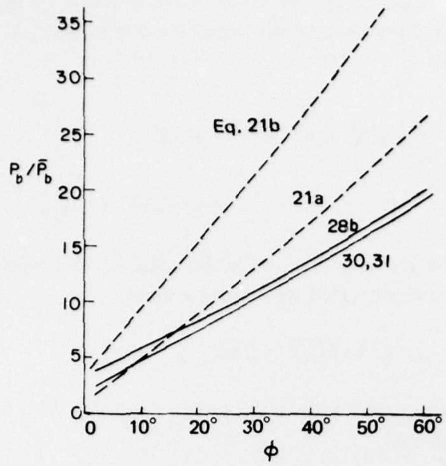


Figure 6. Critical buckling loads for floating wedge. (Note: $\bar{P}_b = b_0\ 2\sqrt{\gamma D}$).

$$\beta(4Db_0\beta^4 - b_0\gamma/3) + \phi^* (2D\beta^4/3 - \gamma/2) = 0.\dagger \qquad (31)$$

This is the equation for the determination of β which appears in eq 30 as an unknown.

Comparison of eq 28b with eq 21a and eq 21b shows that, except for the constant coefficients, the three equations have the same functional form.

In order to compare the approximate expressions obtained for the buckling load P_b, they were evaluated for $E = 30{,}000$ kg/cm^2, $\nu = 0.34$, $\gamma = 0.001$ kg/cm^3, $b_0 = 20$ cm, and $h = 10$ cm. The results are shown in Figure 6.

Proposed method to determine P_{max}

When an ice cover presses against a bridge pier or the leg of a drilling platform, the boundary conditions at the contact area between the ice and the structure fluctuate, depending on a number of factors. Also, the value ϕ is usually not a measurable entity, since according to test observations, at the onset of buckling, the ice cover exhibits many radial (and some nonradial) cracks.

It is therefore proposed to express the largest force an ice cover exerts on an isolated structure (when buckling dominates) by the expression

$$P_{max} = n'D\kappa(\kappa b_0 + m') \qquad (32)$$

where, according to eq 15c and eq 17b,

$$\left.\begin{aligned} D &= \frac{Eh^3}{12(1-\nu^2)} \\ \kappa &= \sqrt[4]{\gamma/4D} \end{aligned}\right\} \qquad (33)$$

and n' and m' are parameters to be determined from field tests.

Since the effective values of E and ν depend upon the nature of the precracked plate, the term $E/[12(1-\nu^2)] = E^*$ should be determined from the same test data. Thus, the proposed expression for P_{max} is

$$P_{max} = n\sqrt[4]{\gamma E^{*3}h^9}\ [\sqrt[4]{(\gamma/E^*h^3)}\ b_0 + m] \qquad (34)$$

where γ is the specific weight of the liquid base, h is the ice cover thickness, and b_0 is the width of pier. The parameters m, n, and E^* are to be determined from appropriate test results.

It is expected that m, n, and E^* will vary for different categories of problems and different ice temperatures (in particular in the vicinity of the melting temperature). This point, as well as the validity of eq 34 to represent P_{max} (when the buckling failure mode predominates), can only be established by comparing eq 34 with the relevant results of laboratory and field tests.

† Note that eq 31 may be obtained directly from the condition $\partial P_b/\partial\beta = 0$.

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